GCE AS/A level

0977/01

MATHEMATICS – FP1
Further Pure Mathematics

A.M. WEDNESDAY, 29 January 2014

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Differentiate $\frac{x}{1+x}$ from first principles. [6]

2. Given that

$$S_n = 1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + \dots + n(n+1)^2,$$

obtain an expression for S_n, giving your answer as a product of linear factors. [6]

3. *(a)* Express $(1 + 2\mathrm{i})^4$ in the form $x + \mathrm{i}y$, where x, y are real. [2]

(b) (i) Hence show that $1 + 2\mathrm{i}$ is a root of the quartic equation $x^4 + 12x - 5 = 0$.

(ii) Determine the other three roots of the equation. [8]

4. The roots of the quadratic equation $2x^2 - 3x + 4 = 0$ are denoted by α, β.
Find the cubic equation whose roots are $\alpha^2\beta$, $\alpha\beta^2$, $\alpha\beta$. [8]

5. The transformation *T* in the plane consists of a reflection in the line $x + y = 0$, followed by a translation in which the point (x, y) is transformed to the point $(x + 1, y + 2)$, followed by a clockwise rotation through 90° about the origin.

(a) Show that the matrix representing *T* is

$$\begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}.$$

[5]

(b) Find the equation of the image under *T* of the line $y = 2x - 1$. [5]

6. *(a)* Use mathematical induction to prove that

$$\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}^n = \begin{bmatrix} 1 & 3^n - 1 \\ 0 & 3^n \end{bmatrix}$$

for all positive integers n. [7]

(b) Determine whether or not this result is true for $n = -1$. [3]

7. *(a)* Given that $\mathbf{A} = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$,

(i) find the adjugate matrix of $\mathbf{A}$,

(ii) find the inverse of $\mathbf{A}$. [5]

(b) **Hence** solve the equations

$$\begin{aligned} 2x + 3y + z &= 13, \\ x + 2y + 3z &= 13, \\ 2x + 3y + 4z &= 19. \end{aligned}$$

[2]

8. The function f is defined by

$$f(x) = \left(\frac{1}{x}\right)^{\sqrt{x}}, \text{ for } x > 0.$$

(a) Show that

$$f'(x) = f(x)\,g(x),$$

where $g(x)$ is to be given in simplified form. [4]

(b) Find the coordinates of the stationary point on the graph of f, giving your answers correct to three significant figures. [3]

(c) Determine the set of values of x for which $f'(x)$ is positive and the set of values of x for which $f'(x)$ is negative. Hence identify the stationary point as a maximum or a minimum. [2]

9. The complex number z is represented by the point *P*(x, y) in the Argand diagram. Given that

$$|z - 2| = 2|z + \mathrm{i}|,$$

(a) show that it can be deduced immediately that the locus of *P* passes through the origin, [2]

(b) show that the locus of *P* is a circle, and find its radius and the coordinates of its centre. [7]

END OF PAPER